EXPERIMENTS WITH A SIMPLE
HURRICANE-INTERACTING OCEAN MODEL

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THESIS

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Hurricane-Interacting Ocean Model

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ABSTRACT

The equations developed by Riehl (1963) for a steady, symmetrical hurricane are used to specify a vortex over a fixed depth boundary layer, which interacts with a simple ocean model. Only vertical mixing in the surface layer is included in the oceanic portion of the model. Interaction of the atmosphere and the ocean occurs through the prediction of boundary layer parameters. Bulk aerodynamic equations are used to calculate the air-sea interaction. Comparison of the predicted boundary parameters with observed mature tropical storms shows good agreement for all variables except the specific humidity, which exceeds saturation. Two attempts to correct for this situation have shown that small changes in the surface moisture and temperature values strongly affect the maximum wind speed predicted. The proportion of the water vapor transported out of the top of the boundary layer without adding latent heat to the boundary layer appears to be an important factor in determining the intensity of a steady-state hurricane.

Although the ocean model is quite simple, realistic horizontal oceanic temperature profiles with respect to the hurricane center were obtained. However the magnitude of changes in oceanic heat content and the sea-surface temperatures were smaller than expected in actual hurricanes, since advection and upwelling effects were not included in the ocean model.

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LIST OF SYMBOLS

Cp	Specific heat of air at constant pressure.
C_D	Drag coefficient.
E	Energy exchange potential.
e_s	Saturation vapor pressure.
f	Coriolis parameter.
HHP	Hurricane heat potential.
L	Latent heat of vaporization of water.
P	Pressure.
Qa	Specific humidity at Ta.
Qe	Latent heat release rate.
Qs	Sensible heat transfer.
Qw	Saturated specific humidity at Tw.
r	Radial distance from center of hurricane.
rh	Radius at which v = 25 m/sec.
ri	Radius of maximum tangential wind.
r_o	Radius from center of hurricane where tangential wind component is equal to zero in the outflow layer.
$\overline{T}$	Mean temperature of ocean layer.
Ta	Atmospheric temperature.
Tae	Pseudo-equivalent temperature.
Tw	Sea surface temperature.
u	Velocity component in x direction
v	Velocity component in y direction.
V_r	Radial wind component.

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V_θ	Tangential wind component.
$V_{\theta h}$	Tangential wind component set at 25 m/sec.
$V_{\theta i}$	Maximum tangential wind component.
ΔP	Thickness of boundary layer.
α	Inflow angle.
θ	Angular measurement.
θ	Potential temperature.
θ_e	Equivalent potential temperature.
θ_{ei}	Equivalent potential temperature at the eye wall.
θ_{eo}	Equivalent potential temperature at r_o.
ρ_s	Density of air.
τ	Frictional surface stress.
ω	Velocity component in vertical direction.

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I. INTRODUCTION

Observational and numerical investigations of tropical
hurricanes have been made by many tropical meteorologists.
Dynamical and structural features of mature three-dimensional
tropical storms are not completely clear but much has been
learned through the use of steady-state axisymmetric hurri-
cane models. Malkus and Riehl (1960) examined the dynamics
and energy transformations in such a steady-state hurricane.

Investigation of inflow trajectories, shown in Figure 1,
gave insight to the oceanic heat source necessary for the
maintenance of a steady-state storm. The dynamic model of
the inflow layer developed useful relations between wind
speed, pressure gradient, surface shearing stress, mass flow
and convergence.

In a more recent work, Riehl (1963) examined some rela-
tions between wind and the thermal structure of the steady-
state hurricane. The steady-state hurricane is defined as
one in which a balance exists between the heat source and
momentum sink at the air-sea interface. In terms of a bulk
aerodynamic estimate of these processes, the heat source de-
pends linearly on wind speed while momentum sink depends on
the square of the wind speed. A steady hurricane model was
developed to compare with observed storms. A two-layer model
was considered with conservation of absolute angular momentum
assumed in the outflow layer, shown in Figure 1, and conser-
vation of potential vorticity assumed in the inflow. With

these assumptions the momentum transport at the air-sea
interface can be specified, and thus the radial profile of
the tangential wind component. By assuming the kinetic ener-
gy necessary to drive a hurricane is provided by the oceanic
heat source, Riehl estimated the heat source required for a
given wind field to exist in steady state.

The purpose of this study is to utilize the steady-state
hurricane model proposed by Riehl in experiments with an
interacting ocean model. Air-sea exchanges beneath the
tropical storm will be examined for a constant depth surface
boundary layer interacting with the ocean surface.

Several numerical simulations of hurricane development
have been made with complicated multi-level models. These
models, which require very short time steps, included some
interaction with the ocean. The scope of the study is to
model only the boundary layer beneath a specified vortex.
The Riehl model is, therefore, used as a diagnostic model
assuming that it applies at each time step. The steady-state
vortex above the boundary layer is defined by Riehl's model,
which is specified by air-sea interaction.

Specification of the boundary layers under the circula-
tion provides an opportunity to study the changes in the
ocean as well as the vortex. An essential assumption carried
over from Riehl was a steady, symmetrical vortex which in
early experiments, is stationary. Leslie and Smith (1970)
point out that the importance of the surface boundary layer
on the dynamics of a hurricane is due to the meridional

circulation of the vortex, providing a strong coupling be-
tween the azimuthal and vertical velocity fields. However
this study will use the simple boundary layer of Riehl.

Several investigations have shown that areas of low sea-
surface temperatures occur in the wake of hurricanes. Jordan
(1964) analyzed sea-surface temperature for the months of
typhoon seasons and reported marked cooling of the ocean
surface with passage of the hurricane. Leipper (1967)
analyzed oceanic temperature for hurricane Hilda of 1964.
In the latter study, comparison of pre-storm merchant ship
reports and post-storm data, gathered by a planned cruise of
the area in the Gulf of Mexico where Hilda passed, indicated
significant cooling in the surface layer during the passage.
Jordan suggested that cooling of the sea-surface temperature
was due, in part, to upwelling of cooler water or vertical
mixing caused by the winds. Jordan concluded that mixing
was the more important factor. However, other authors such
as Fisher (1958) have considered upwelling to be the most
important. Leipper (1967) stated that in addition to up-
welling of cooler waters and vertical mixing of the surface
layer, horizontal advection of surface waters and the thermal
transfer of energy from the ocean to the atmosphere are im-
portant factors in determining the observed oceanic thermal
structures. Leipper concluded that the dominant process in
the observed cooling is the Ekman suction resulting from
wind mixing and that upwelling is the next most important.
A theoretical two-layer ocean model developed by O'Brien and

Reid (1967) to simulate circulation beneath a hurricane verified that upwelling was an important factor.

In this research the oceanic model is an elementary attempt to simulate the ocean temperature field resulting from vertical mixing due to the air-sea exchange. Eventual refinements of the ocean model may include upwelling of cooler water and oceanic advection, and also movement of the hurricane over the ocean will allow prediction of the oceanic effects. Volgenau (1970) has incorporated the oceanic effect in terms of the hurricane heat potential (HHP) which is defined as the excess of heat content over that of water at 26C.

Those specific quantities examined in this study are the temperature and moisture fields of both the atmosphere and the ocean, the effects of the atmosphere upon the oceanic fields and vice-versa. Also the effect of varying initial maximum wind speeds will be examined. Eventual study will include the effects of a varying drag coefficient in the boundary layer. In general, this study is an attempt to create an interacting model of a hurricane boundary layer and a simple ocean model, which may be developed into more detailed descriptions of the effects of sea-air interaction in hurricanes.

II. DISCUSSION OF THE MODEL

This study is an experiment with the steady-state, symmetrical hurricane as described by Riehl (1963); therefore this discussion will follow relevant development in his paper.

A symmetric, 17x17 rectangular grid is used in Cartesian coordinates with 15 nautical mile spacing. The center of the hurricane coincides with the center of the grid and the eye wall is linked to the first grid point outbound along a diagonal. Cartesian coordinates are used to link the symmetrical hurricane to the model of an ocean within a rectangular region. In the initial experiments the hurricane is stationary, but eventually the hurricane will be moved with respect to the ocean model in a Cartesian coordinate system. Transformation from the arbitrary Cartesian grid to polar coordinates centered on the eye of the hurricane is part of the procedure. (See Fig. 2)

A. REVIEW OF RIEHL'S MODEL

1. Outflow Layer

The initial parameters include the maximum tangential wind component, $v_{\theta i}$, the radius of maximum wind, r_i, along with the latitude of the storm. Riehl and Malkus (1961) have shown that conservation of absolute angular momentum is a reasonable first approximation in the outflow layer. (See Fig. 1) This constraint leads to the following

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expression for computing r_o, the radius where $v_\theta = 0$,

$$r_o = (2 v_{\theta i}' \, r_i / f)^{1/2},$$ (1)

where f is the coriolis parameter. It is noted that by specifying the eye wall or maximum wind at a diagonal grid point, r_o becomes a function of the grid spacing. Variables which are dependent on radius only are computed along a diagonal. Those variables include pressure, vertical motion, temperature and moisture advection, and large scale moisture flux.

2. Inflow Layer

Conservation of potential vorticity in the inflow layer was also demonstrated by Riehl and Malkus (1961). Matching the outflow and inflow layer at the eye wall leads to the relationship,

$$v_\theta' \, r^{1/2} = Const,$$ (2)

which is a good approximation to v_θ-r profiles found in mature hurricanes (see examples in Riehl, 1963). Inside the eye wall, solid body rotation is assumed to determine v_θ. Thus given $v_{\theta i}$ and r_i, the tangential component of the wind can be determined from Eq. (2) at any radius. In particular the tangential wind at the radius r_o is used in the calculation of the initial radial component of the wind.

Integration of the momentum equation over a shallow inflow layer of thickness ΔP where potential vorticity is assumed to be conserved, results in the following expression

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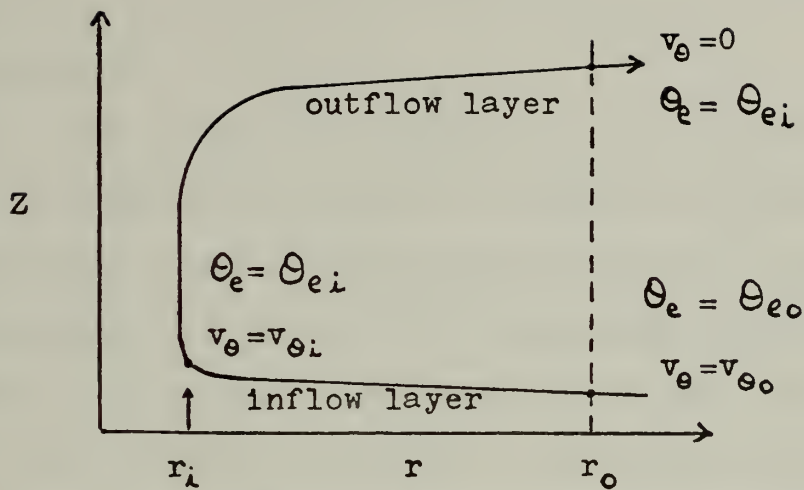
Illustrating hurricane mass circulation
v_θ is tangential wind component, Θ_e is
equivalent potential temperature, r is
radius

Figure 1
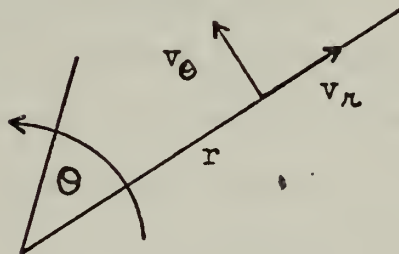
Illustrating polar coordinate system used
v_θ is tangential wind component, v_r is
radial wind component, r is radius, Θ is
angular measurement

Figure 2

for the radial wind component,

$$v_r = g/\Delta P \, r_0 \, v_{\theta_0}^2 / v_{\theta_0} \left(\frac{r_0}{r}\right)^{1/2} + 2fr, \tag{3}$$

where g is the acceleration of gravity and ΔP is the inflow thickness set at 100 mb.

The tangential component at the surface (v_{θ_0}) must also be specified to define a complete surface wind for calculation of the air-sea boundary fluxes. Therefore v_{θ_0} was assumed to be 0.7 of the wind speed at the top of the boundary layer, which was specified by Eq. (2). The use of this coefficient is equivalent to specifying an effective drag coefficient based on the tangential component at the top of the boundary layer. Choosing a value of 0.7 reduces the drag coefficient (C_D) by a factor of 0.49 for momentum exchange, and by 0.7 for the heat and moisture exchanges. As noted in the introduction the heat source depends linearly on wind speed while momentum sink depends on the square of the wind speed.

3. The Steady State Value of V

Riehl proposed that a particular relation exists between the heat source and momentum sink in the interior of hurricanes, if a steady state velocity field is observed. This proposal leads to the following expression for the maximum wind speed,

$$v_{\theta i}^2 \, (m/sec) = 10^2 \, E \left[L_N \frac{r_h}{r_i} + \frac{4 f r_h}{3 \, v_{\theta h}} \right] \tag{4}$$

which satisfies both the tangential distribution from

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momentum and from heat source considerations, where r_h is the point where $v_{\theta_h} = 25$ m/sec. as chosen by Riehl. An energy exchange potential parameter, E, is defined as,

$$E = (T_w - T_a) + L/C_p (Q_w - Q_a) \tag{5}$$

where Tw and Qw are the temperature (C) and the saturated specific humidity (gm/kgm) at the sea surface and Ta and Qa are the temperature and specific humidity of the atmosphere in the boundary layer. L represents the latent heat of vaporization (597.3 cal/gmw) and Cp is specific heat constant for (.24 cal/gm deg).

B. PREDICTION EQUATIONS

1. Atmospheric Prediction

In the atmosphere the predicted variables are the potential temperature and the specific humidity above the sea surface. These two parameters, along with pressure, determine the equivalent potential temperature θ_e. Riehl (1963) has shown that θ_e is one factor in determining the maximum wind.

The thermodynamic equation is used in the following form,

$$\frac{\delta \theta}{\delta t} = - \left[\frac{\delta N_\theta \, \theta}{r \delta \theta} + \frac{\delta N_r \, r\theta}{r \delta r} + \frac{\delta \omega \theta}{\delta P} \right] + Q_s \tag{6}$$

where θ is the potential temperature (K), ω is the vertical motion (mb/hr), P is pressure (mb) and Qs is sensible heat transfer. The first term on the right hand side of Eq. (6) is neglected due to symmetry. Potential temperature is

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assumed to be constant in the boundary layer and thus could be created from the air temperature (Ta) field knowing the surface pressure. The effect of expansion cooling is included by using potential temperature in the thermodynamic equation.

Moisture changes were determined in similar fashion using,

$$\frac{\delta Q_a}{\delta t} = -\left[\frac{\delta \acute{v_\theta} \, Q_a}{r \, \delta \theta} + \frac{\delta \acute{v_r} \, r \, Q_a}{r \, \delta r} + \frac{\delta \omega Q_a}{\delta P}\right] + Q_e, \quad (7)$$

where Qa is the specific humidity (gm/kgm) and Qe is the latent heat transfer. The first term on the right hand side is neglected because of symmetry.

Because the ocean model is in Cartesian coordinates, surface exchange values are computed in rectangular coordinates. The transfer of sensible heat per unit volume is computed with the bulk transport equation in the form,

$$Q_s / C_p = \rho_s \, C_D \, (T_w - T_A) \, \sqrt{u^2 + v^2}, \quad (8)$$

in units of cal/sq cm sec where Cp is specific heat, ρ_s is the atmospheric density above the sea surface, Tw is the temperature of the ocean, Ta is the atmospheric temperature, and $\sqrt{u^2 + v^2}$ represents the total wind speed. Latent heat transfer or the rate of moisture input per unit volume is calculated by,

$$Q_e / L = \rho_s \, C_D \, (Q_w - Q_a) \, \sqrt{u^2 + v^2}, \quad (9)$$

in units of cal/sq cm sec where L is the latent heat of

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vaporization, Qw is the specific humidity at the tempera-
ture of the ocean surface (Tw) and Qa is the specific
humidity of the atmosphere. The Bowen ratio is defined as
the ratio of sensible heat to latent heat exchange and is
an indication of the relative importance of these two
mechanisms of energy transfer from the ocean to the atmos-
phere.

For use in Eq. (8) and (9), the wind is delineated
into u and v Cartesian components. Cartesian components are
transformed from the radial and tangential components of the
wind as follows,

$$u = \mathcal{N}_r \cos \Theta - \mathcal{N}_\theta \sin \Theta$$
$$\mathcal{N} = \mathcal{N}_r \sin \Theta + \mathcal{N}_\theta \cos \Theta \,, \tag{10}$$

where the angle Θ increases in a counter-clockwise direction
from the x-axis so that u is positive for westerly winds and
v is positive for southerly winds.

The equation of motion in polar coordinates, neg-
lecting lateral friction, was used to determine the radial
component of the wind, v , which was in balance with the new
tangential wind component obtained from the following
equation,

$$\frac{d\mathcal{N}_\theta}{dt} + \frac{\mathcal{N}_\theta \mathcal{N}_r}{r} + f\mathcal{N}_r = -\frac{1}{\rho} \frac{\partial P}{r \partial \theta} \quad \frac{1}{\rho} \frac{\partial \Upsilon}{\partial z} \,, \tag{11}$$

In pressure coordinates, assuming a steady state, Eq. (11)
yields,

$$\mathcal{N}_r = \frac{-g \frac{\partial \Upsilon}{\partial P}}{f + \frac{\partial \mathcal{N}_\theta}{\partial r} + \frac{\mathcal{N}_\theta}{r}} \,, \tag{12}$$

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where γ is the wind stress, $\partial P / \partial \theta$ is zero due symmetry of
the storm, $\omega \frac{\partial N_\theta}{\partial P}$ is zero because ω is zero at the surface
and $\frac{\partial N_\theta}{\partial P}$ is assumed zero at the top of the boundary layer.
A boundary layer factor of 0.7 was incorporated to account
for the vertical profile, as was described in the inflow
layer section, II A. The surface stress is defined to be
proportional to the square of the surface wind,

$$\gamma_{\theta s} = C_D \, \rho_s \, N_{\theta s}^2 / \cos \alpha , \qquad (13)$$

where α is the inflow angle measured relative to isobars.
The stress is assumed to be zero at the top of the boundary
layer.

Computations of vertical motion and advection in
Cartesian coordinates did not prove to be successful. This
was because any values such as vertical motion or the ad-
vection quantities, which involve x and y derivatives were
not symmetrical with respect to the diagonals. Therefore,
asymmetries resulted in the temperature and moisture fields
and prevented long period predictions. Transforming to a
polar coordinate system for the computation of vertical
motion and the advection quantities conserved the symmetry
of the initial fields. Also, for asymmetric temperature
and moisture fields, a small error is introduced by the
neglection of the azimuthal advection. Due to the large
tangential velocities in a hurricane, various properties
tend to be symmetric with respect to the center and omitting
the azimuthal advection introduces a much smaller error than
the use of Cartesian coordinates.

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Vertical motion at the top of the boundary layer was computed from the continuity equation, in polar coordinates,

$$\frac{\partial N_\theta}{r\,\partial\theta} + \frac{\partial r\,N_r}{r\,\partial r} + \frac{\partial \omega}{\partial P} = 0 ,\qquad(14)$$

where the change of the tangential wind with respect to angular change is zero in a symmetric storm. Vertical motion at the surface is assumed to be zero and the boundary layer is chosen to be 100 mb thick as an approximation to the first kilometer of a tropical atmosphere.

2. Oceanic Prediction

Predicted variables in the oceanic portion of the model are the sea-surface temperature (Tw) and the saturated specific humidity (Qw) at temperature Tw. A crude approximation of the oceanic energy loss centers on the concept of hurricane heat potential, HHP, defined by Volgenau (1970) as the excess of heat content over that of water at 26 C. A temperature profile as shown

$$\begin{array}{c} 28^\circ \quad 30^\circ \\ \text{100 m} \quad \overline{T} = 29^\circ \\ 28^\circ \end{array}$$
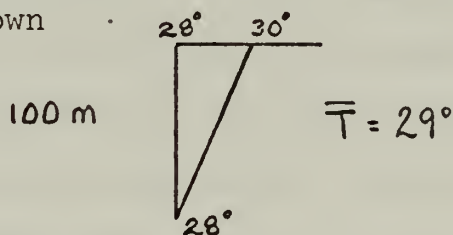
defines a hurricane heat potential of 3×10^4 cal/sq cm by,

$$HHP = C_P\,(\overline{T} - 26) \times depth ,\qquad(15)$$

where $\overline{T}$ is the mean temperature of the chosen ocean layer with a depth of 100 meters. The loss of energy by the ocean or the change in heat potential is equal to the sensible and latent transfer of energy during each time step,

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that is,

$$\Delta HHP = \Delta t \times 3600.0 \times (Q_s + Q_e) ,$$ (16)

where Δt is the prescribed time step. After a time step
the reduced magnitude of heat potential corresponds to a
new mean temperature in the upper layer of the ocean and
thus a sea-surface temperature can be calculated. This
scheme accounts for sensible heating and evaporation but
neglects radiation gains, horizontal divergence or upwelling.
Since the saturation specific humidity field is computed
from the Clapeyron equation, the evaporation from the sea
surface is decreased as the surface temperature is de-
creased.

C. INTERACTION BETWEEN THE ATMOSPHERIC AND OCEAN MODELS

In the atmospheric model the maximum wind speed deter-
mined by the heat source is a function of the energy ex-
change potential, E, as given by Eq. (4). A new mean value
of $\bar{E}$ was calculated after each time step using the newly
predicted atmospheric and oceanic parameters. However the
ocean is effectively an infinite heat source and the result
of this approach was to bring the atmosphere into equilib-
rium with the ocean, that is, E tended to zero. Consequent-
ly the maximum wind speed predicted from Eq. (4) also
decreased to zero although very warm and moist air was
available in the atmospheric boundary layer.

In a second approach, the maximum wind speed was calcu-
lated from the equivalent potential temperature at the eye

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wall as follows,

$$V_{\theta i} \, (m/sec) = 14.1 \, (\theta_{ei} - \theta_{eo})^{1/2} , \qquad (17)$$

where θ_{eo} is set at 350 K. The tangential wind component
was computed at other grid points by the v_θ-r relationship
as given by Eq. (2).

D. INITIALIZATION OF SURFACE TEMPERATURES AND HUMIDITIES

Integration of the hydrostatic equation for typical
values of θ_e in a hurricane produces a relation,

$$P' = -2.56 \, \theta_e' , \qquad (18)$$

where P' is the pressure departure from 1005 mb and θ_e' is
the equivalent potential temperature departure from 350 K.
This expression specifies eye wall pressure variation caused
by the oceanic heat source. With the use of Eq. (17) and
(18), the pressure and equivalent potential temperature at
the eye wall may be specified for a given maximum wind
speed. The pressure distribution along each diagonal was
then computed from the gradient wind equation. The pressure
at intermediate points throughout the symmetric field was
determined by linear interpolation using pressure at the
eye wall and the distribution of v_θ with radius.

Initialization of the temperature and moisture fields
both in the boundary layer and at the ocean surface are
important parts of this model. In the initial experiments
the sea-surface temperature, Tw, was initialized at a con-
stant value of 30 C throughout the grid. The available

moisture at the ocean surface, Qw, was calculated from the integrated form of Clapeyron's equation for the saturation vapor pressure over water,

$$\text{Ln}\frac{e_s}{6.105} = 25\frac{T-273}{T} - 5.2\,\text{Ln}\frac{T}{273}, \tag{19}$$

where e_s is the saturation vapor pressure. Assuming the air in contact with the water is saturated, specific humidity was calculated from,

$$q_s = \frac{.622\,e_s}{P - .373\,e_s}, \tag{20}$$

in units of gm/kgm.

Two different methods were used for initialization of the air temperature and moisture fields within the boundary layer. One approach was to specify the initial values of the air temperature, Ta, and the specific humidity, Qa, in terms of the equivalent potential temperature at the eye wall which is related to the maximum tangential wind in Eq. (17). Specification of initial parameters at the eye wall from equivalent potential temperature is consistent with the scheme used in predicting the new maximum wind speed. This method involved the solution of two simultaneous equations defining E and the pseudo-equivalent temperature. These equations were,

$$Q_a = Q_w - C_P/L\left[E - (T_w - T_a)\right], \tag{21}$$

and,

$$Q_a = T_a\frac{C_P}{L}\,\text{Ln}\frac{T_{ae}}{T_a}, \tag{22}$$

where Tae is the pseudo-equivalent temperature. Following
Riehl, it was assumed that E may be treated as constant
over the inner region of a hurricane and the equivalent
potential temperature difference between the eye wall and
the outer regions of the storm is constant over the boundary
layer depth Δz . Thus the expression for the effect of the
total heat source in increasing the equivalent potential
temperature is,

$$\Theta_{ei} - \Theta_{eh} = \frac{C_D}{\Delta z} E \int N_0 \, dt ,\qquad (23)$$

Effectively this expression implies bulk aerodynamic trans-
fer of heat and moisture in a fixed boundary layer. The
sensible and latent heat transfer where $r < r_h$ is crucial in
determining the increased equivalent potential temperature
required to generate hurricane force winds. The value of E
was computed from Eq. (4). The adiabatic equivalent tempera-
ture is related to the equivalent potential temperature by,

$$\Theta_e = T_{ae} \left(\frac{1000}{P} \right)^{0.286} .\qquad (24)$$

By specifying the initial state from the equivalent poten-
tial temperature at the eye wall, the effect of expansion
cooling and sensible heat source at the surface are taken
into account. Riehl suggested that Θ_e must rise above
350-352K to allow deep convection which warms the column
and lowers the central pressure to values which can sustain
hurricane force winds.

Solution of Eqs. (23) and (24) gives an initial value
of surface temperature and moisture at the eye wall.

Therefore expression Eq. (17) relating $v_{\theta i}$ to Θ_{ei} applies only at the eye wall unless these values are specified elsewhere. To initialize the remainder of the Ta and Qa values it was assumed that the relative humidity varied linearly between the value determined at the eye wall and a value at the outer radius. The latter value was selected to be typical of undisturbed tropical air. For the remaining grid points simultaneous solution of the energy exchange potential, E, equation and the Clapeyron equation including the specified relative humidity was used to determine the initial state.

The present experiment has shown that a second approach may be as effective as the previous solution of simultaneous equations. By initializing the atmospheric temperature at an arbitrary value, say equal to the oceanic value, while the moisture was made consistent with the maximum wind speed through the equivalent potential temperature, it was found that the model came into balance with a steady-state hurricane after 10 hours. Comparison of the temperature and humidity fields produced by both approaches shown in Fig. 3 and 4, demonstrates this favorable result.
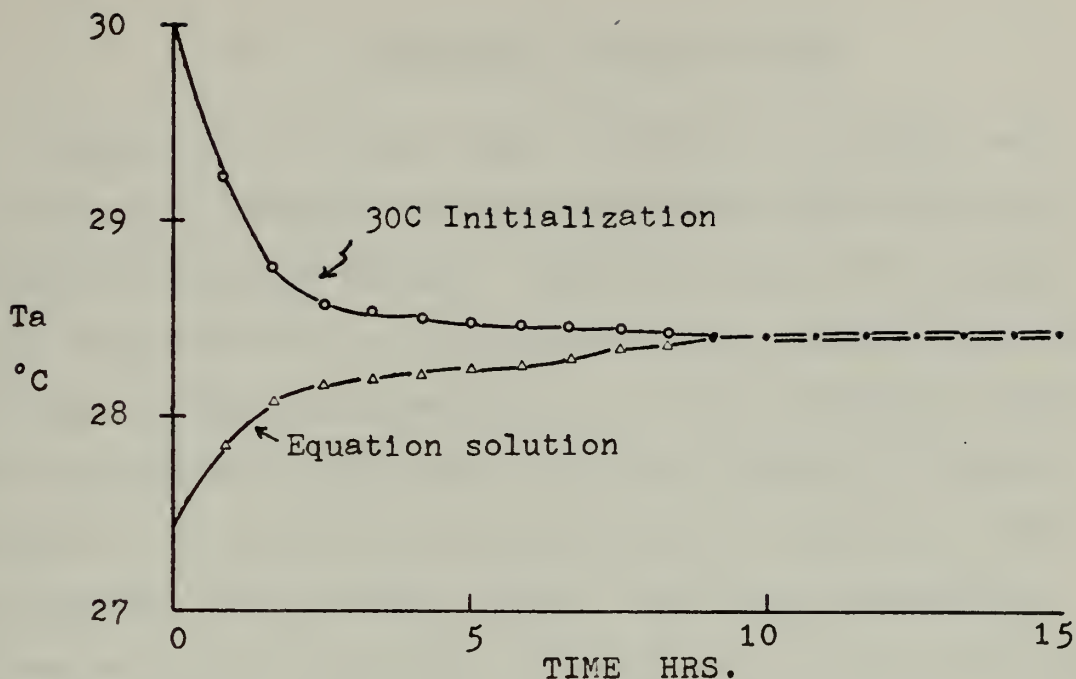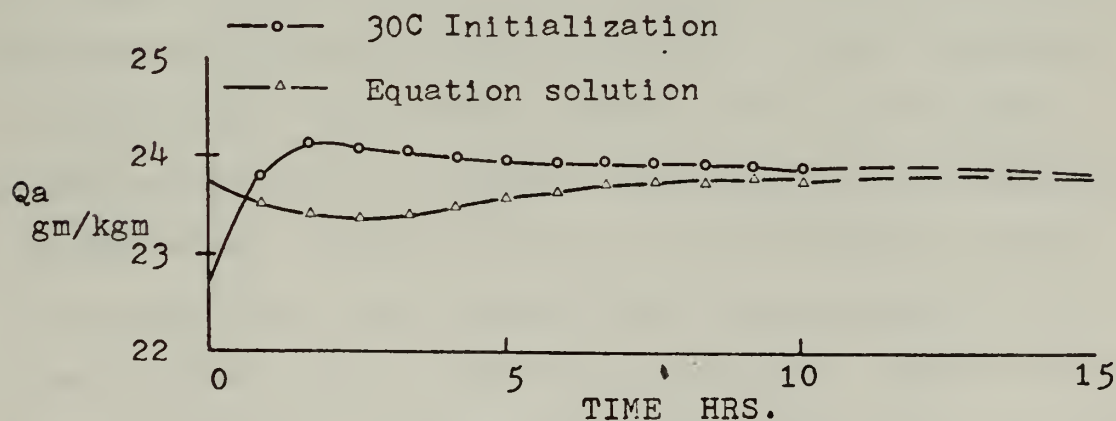
Evolution of atmospheric temperature at the eye wall for
two initialization procedures. (See text.) Equation
solution refers to the simultaneous solution of Eqs. 23
and 24.

Figure 3

Same as Fig. 3 except for specific humidity at the eye
wall.

Figure 4

III. ATMOSPHERIC MODEL PROFILES

Results of the basic model described in earlier sec-
tions may be compared with the observed surface structure
of mature tropical storms. Specifications of the tangen-
tial wind profile, v_θ , according to Eq. (2) has been shown
by Riehl (1963) to be realistic, yet the remaining variables
of the predicted hurricane need to be verified. Observa-
tions of the surface characteristics of a hurricane have
been summarized by Riehl (1954). For these comparison
experiments the sea-surface temperature, Tw, was held fixed
to model the steady-state atmospheric circulation.

The effect of the surface stress is to develop a cross-
isobaric flow, the radial component of the wind (v_r), shown
in Figure 5. The radial wind component increases toward
the eye wall, with a maximum value of v_r which compares
favorably with the velocity profile for a steady-state
hurricane described by Riehl. Vertical motion is determined
by the radial gradient of r v_r in an axisymmetric storm.
All values of ω were negative with the strongest ascent
concentrated near the center of the storm. Figure 6 illus-
trates the area of strong upward vertical motion resulting
from the inflow near the eyewall. Note that no strong radi-
al inflow bands, which would represent the satellite-
observed cloud formations spiralling toward the hurricane
center, can exist in the axisymmetric vortex.
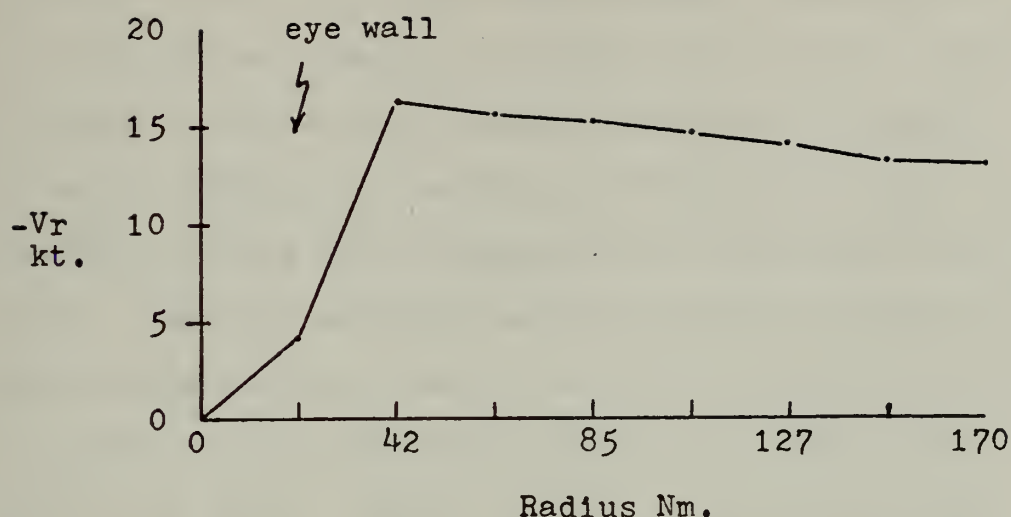
Typical profile of the radial component of wind for
the basic model beginning from initial maximum wind
of 110 kts.

Figure 5
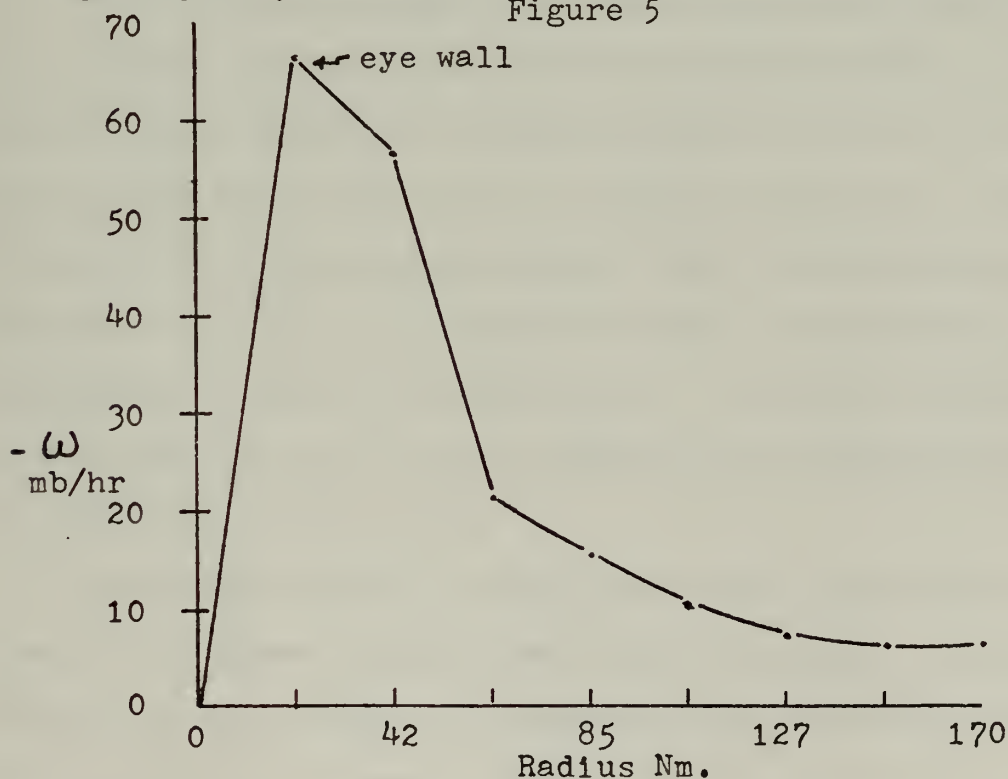
Same as Fig. 5 except for vertical motion in pressure
coordinates.

Figure 6

Although potential temperature of the surface air increases along the inflow trajectories, the surface temperature decreases toward the storm center until a balance of expansion cooling and sensible heating is reached. The expected temperature pattern is verified by Fig. 7 with the maximum cooling of the surface air occurring at the eye wall. Specific humidity may be expected to increase along the inflow trajectories as the air in contact with the ocean acquires moisture through evaporation, which should increase with wind speed. Figure 8 shows a slight increase of moisture content predicted as the air moves inward.

It has been observed that precipitation in hurricanes is strongly concentrated near the inner core of a storm. Riehl (1954) notes the average rainfall rate at a fixed point will be on the order of 15-20 cm per day. For comparison the large scale moisture flux due to vertical motion through the top of the boundary layer was converted to an equivalent depth of precipitation. The model predicts only 3-4 cm per day, but Fig. 9 verifies the concentration of rainfall at the eye wall.

The surface layer air acquires both latent and sensible heat as it moves inward toward lower pressure. These results show that the transfer of sensible heat from the ocean to the atmosphere is small compared to the latent heat transfer. In the initial state the air temperature, T_a, is approximately equal to the sea-surface temperature, T_w, and the atmospheric specific humidity, Q_a, is small compared to the
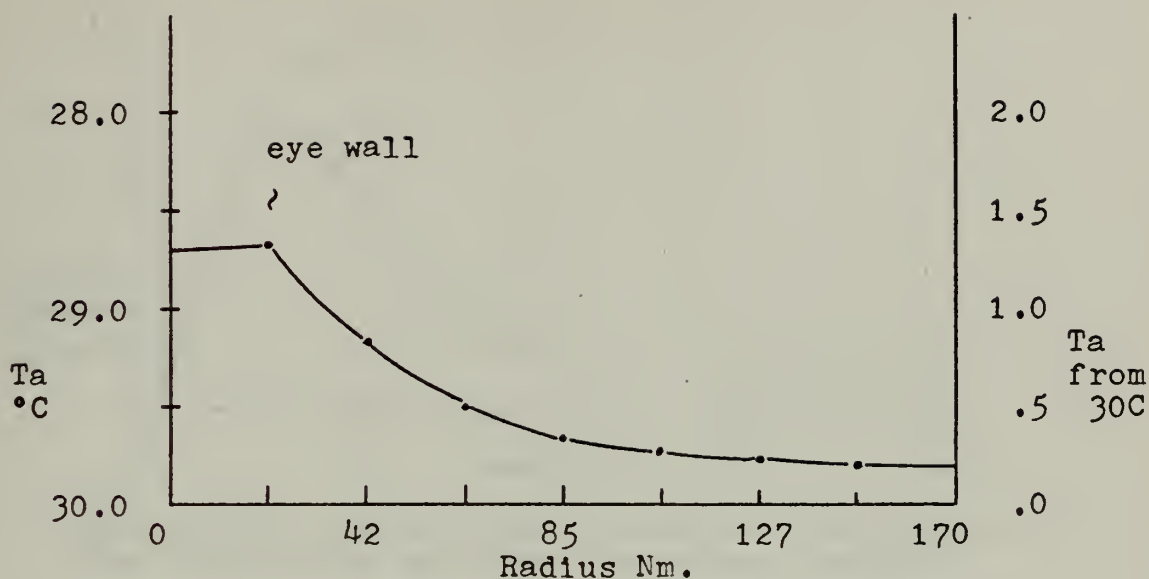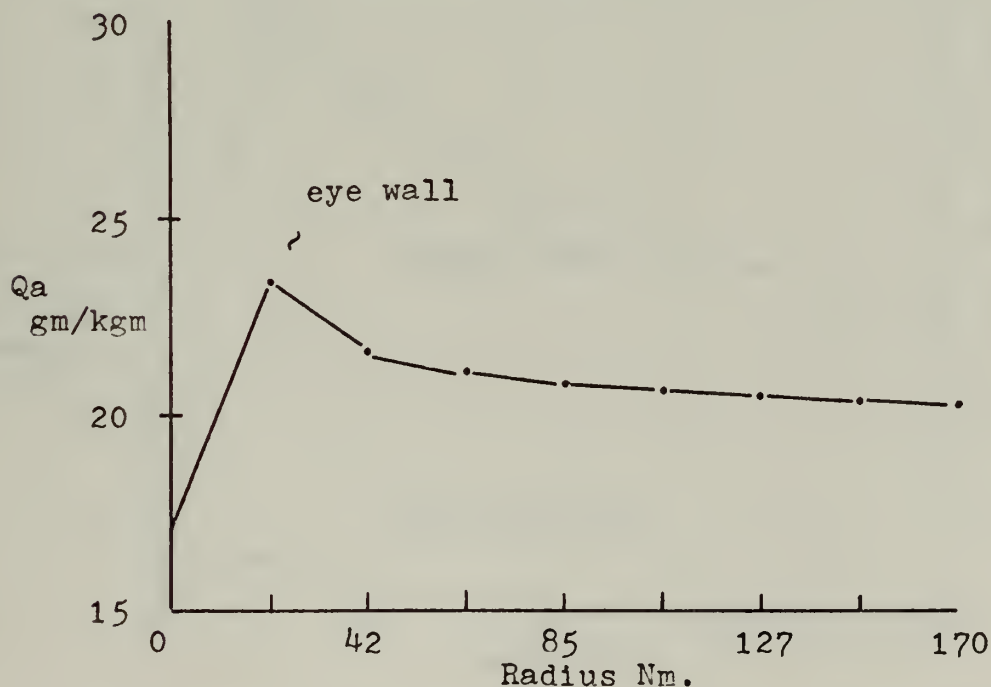
Same as Fig. 5 except for atmospheric temperature.
Right hand scale represents the difference between
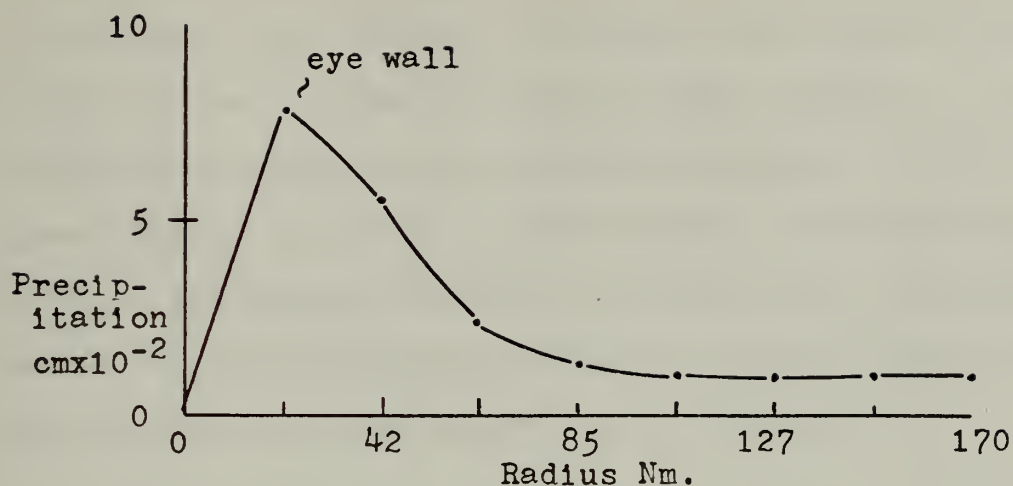oceanic and atmospheric temperature.

Figure 7

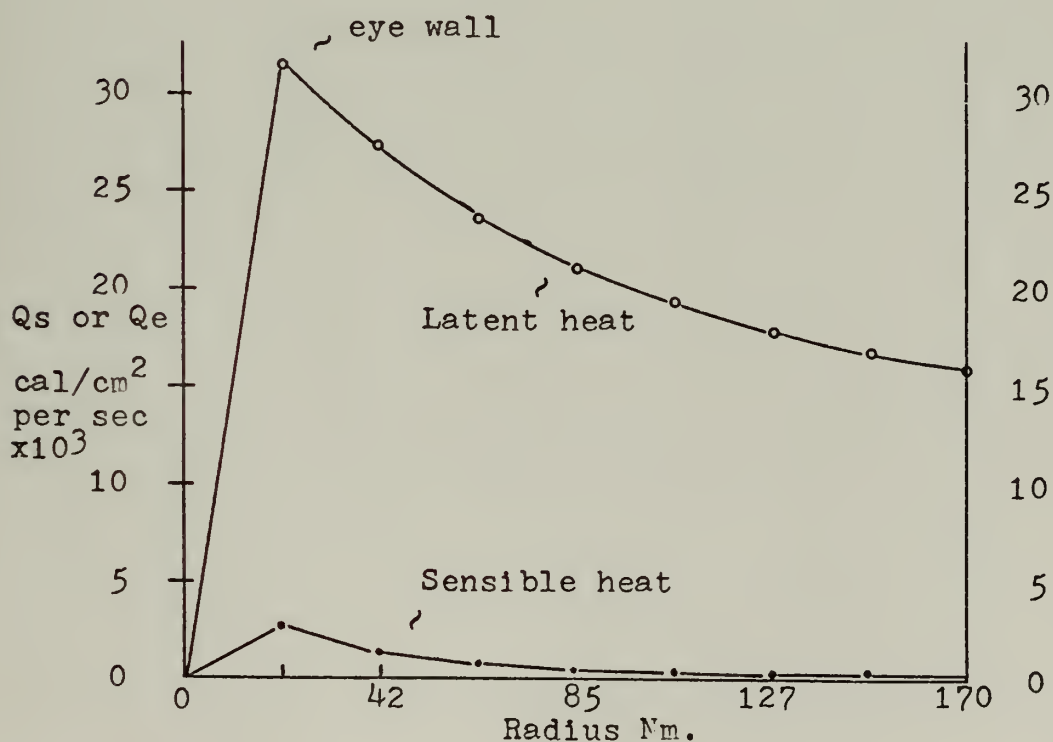Same as Fig. 5 except for specific humidity.

Figure 8

Same as Fig. 5 except for accumulated precipitation from large scale moisture flux after one hour.

Figure 9

Same as Fig. 5 except for sensible heat (Qs) and latent heat (Qe) exchange for the surface. Ocean temperature is fixed at 30C.

Figure 10

saturation specific humidity at the ocean interface. Therefore latent heat transfer is larger than sensible heating. Figure 10 is a comparison of the heat transfer. The large addition of moisture due to evaporation may either be precipitated or stored in the column. The large precipitation at the eyewall is due to the fact that the moisture which is not precipitated along the radial inflow trajectory will be carried inward to the eye wall.

IV. MODIFICATION OF THE MODEL

An encouraging aspect of the initial experiments was
obtaining the expected atmospheric horizontal temperature
profile. However, prediction of moisture content repre-
sented a major shortcoming of the initial model. That is,
the advection scheme plus the sensible heat exchange pro-
duced the expected temperature profile, but advection plus
evaporation produced an excessive increase in moisture in
the boundary layer. As shown in Fig. 12, surface relative
humidity values at the eye wall exceeded saturated condi-
tions after short time periods. Either the moisture trans-
port due to vertical motion in Eq. (7) was not removing
enough moisture from the boundary layer, or the evaporation
from the sea was too large.

A possible explanation may be that the surface exchange
of moisture was overestimated. Robinson (1966) reports
that the drag coefficient, C_D, for the sensible and latent
heat exchange at high wind speeds is less than the drag
coefficient for momentum exchange by a factor of one-third.
In the momentum exchange at the sea surface the pressure
forces dominate over molecular processes at the interface.
However near the air-sea interface molecular processes
determine the rate of transport of heat and water vapor.
Thus the transfer coefficient for momentum may differ from
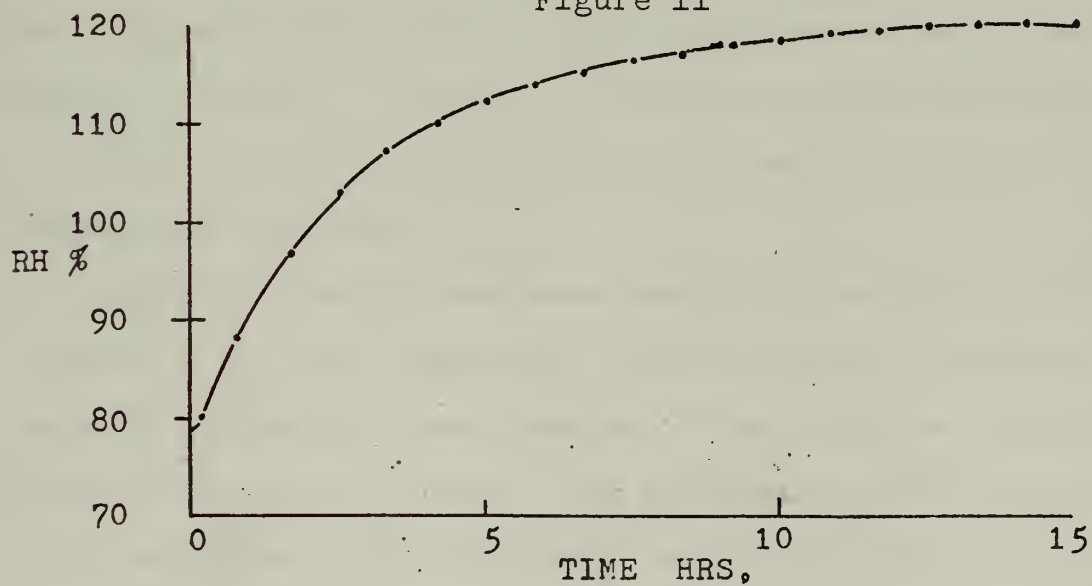that for sensible heat and moisture exchanges which are

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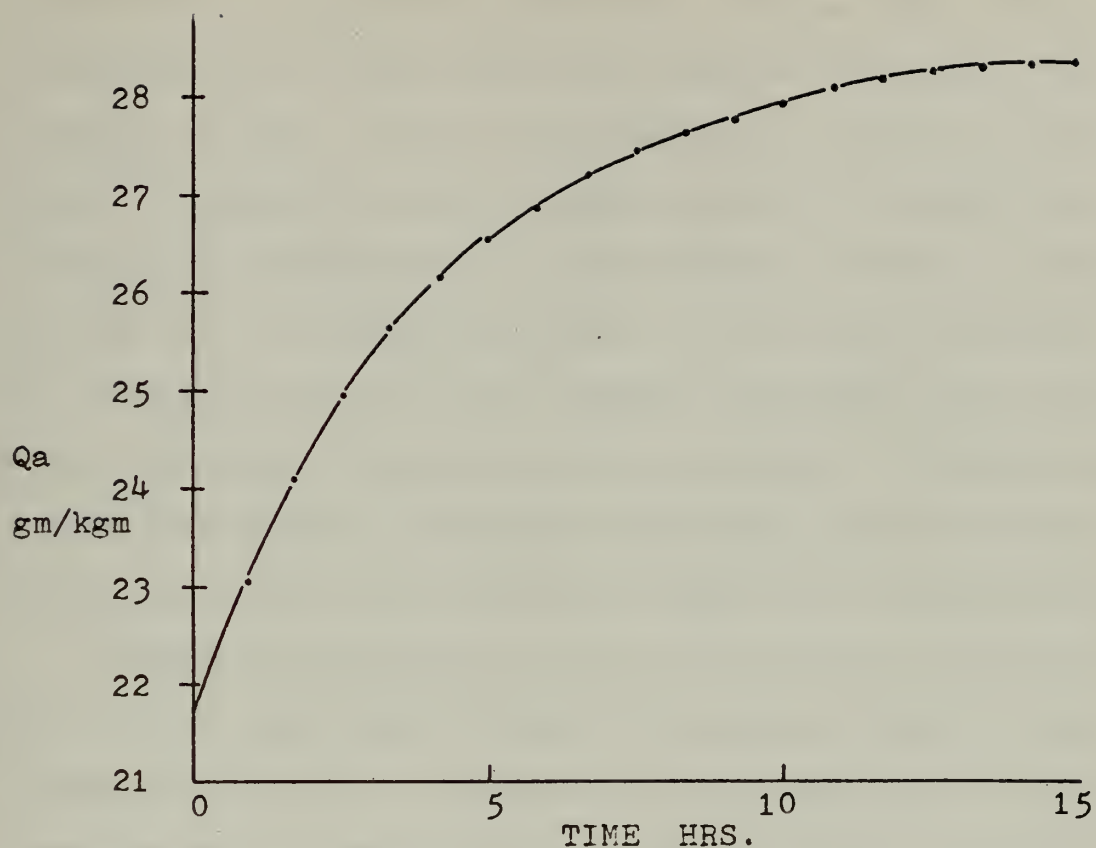
Evolution of specific humidity at the eye wall for the same experiment as described in Fig. 5.

Figure 11

Evolution of relative humidity at the eye wall for the same experiment as described in Fig. 5.
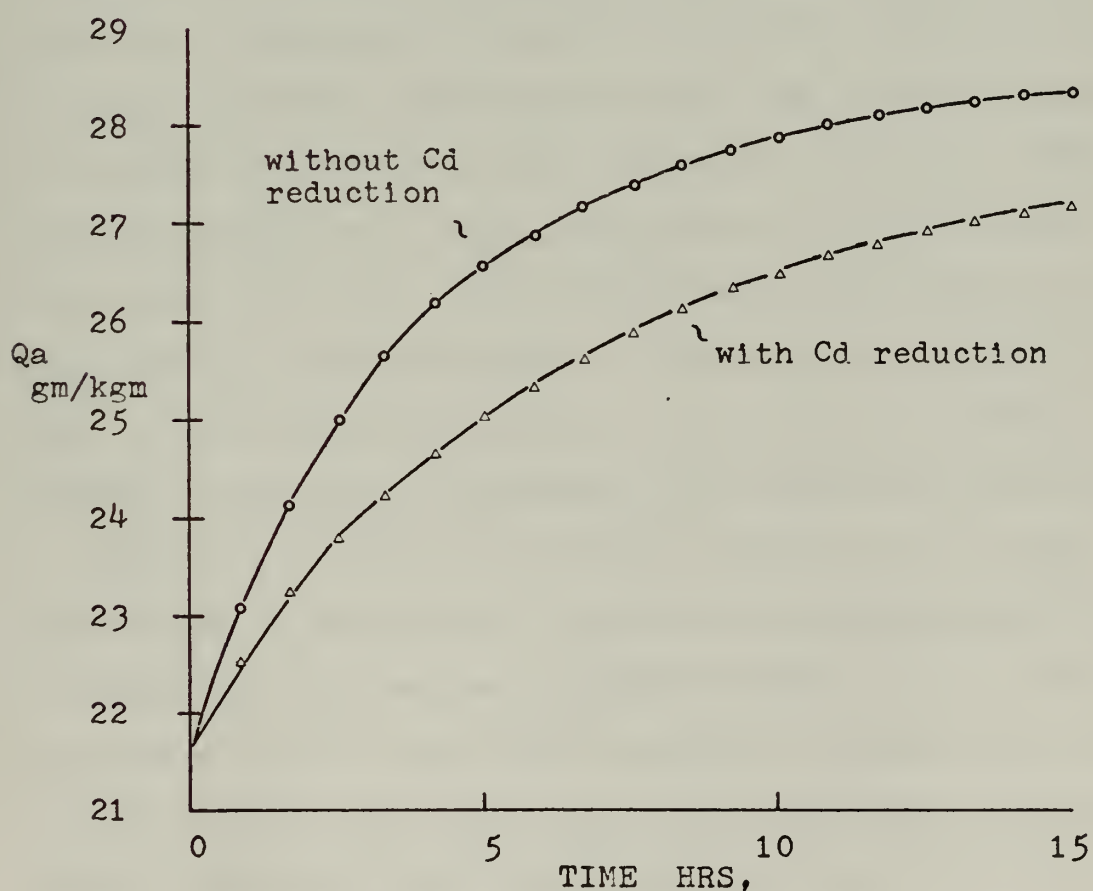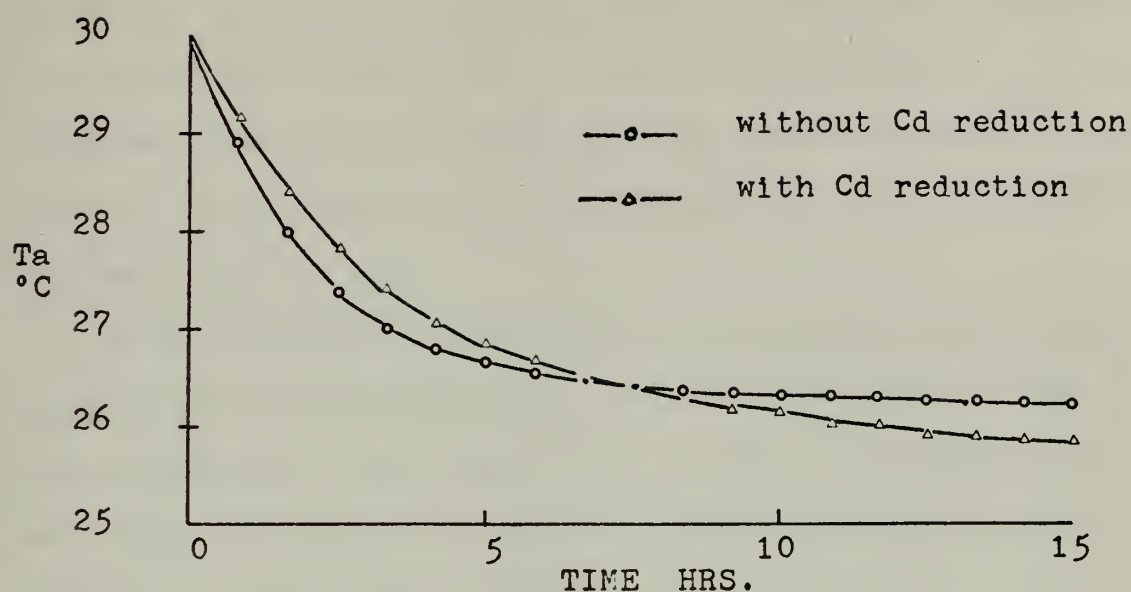
Figure 12

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restricted by molecular diffusion. To test this effect,
computations were made with the sensible and latent heat
exchange in Eqs. (8) and (9) reduced by a factor of one-
third. Taking this into account served to decrease the
surface air temperature and the moisture content as shown
by Fig. 13. It should be noted, if the drag coefficient
for momentum had also been reduced, the radial and vertical
wind components would have been decreased. A reduced hori-
zontal and vertical transport would have resulted which
would have offset the favorable reduction in evaporation.

Another explanation for the accumulation of moisture
may be that the model includes no representation of the
cumulus clouds or "hot towers" that characterize a hurri-
cane eye wall. Such cumulus convection would transport
additional moisture through the top of the boundary layer.
Two approaches were used to reduce the excessive accumula-
tion of moisture. The drag coefficient reduction suggested
by Robinson was included in all experiments comparing these
convective mechanisms.

Convective mechanisms were based upon relative humidity
criteria. In each time step, a small amount (initially 0.5
percent) of moisture was removed if the relative humidity
exceeded a critical value. The amount of moisture removed
could be varied with increasing relative humidity to achieve
a balance between the moisture increase due to the hori-
zontal advection and the convective moisture reduction.
The amount of moisture removed by this scheme was accumu-
lated and converted to an equivalent depth of precipitation.

Evolution of atmospheric temperature (upper diagram) and specific humidity (lower diagram) with time for experiment with and without Robinson's C reduction.
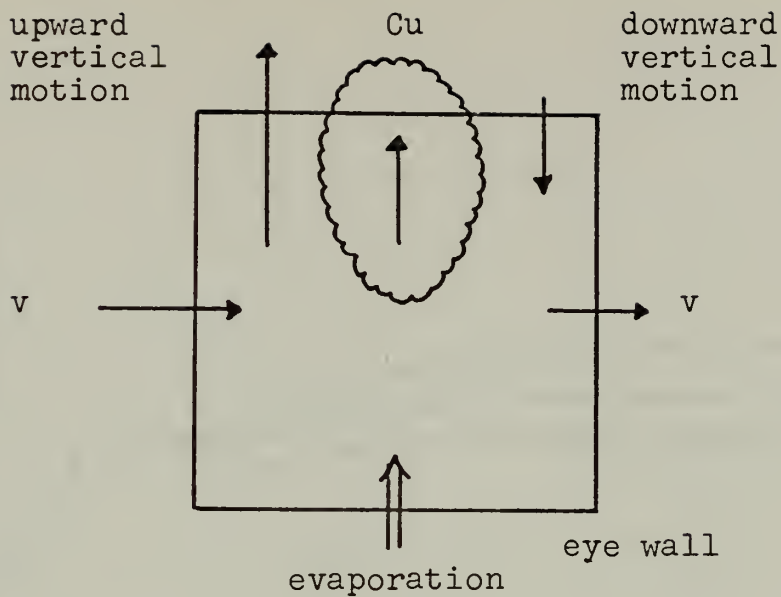
Figure 13

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The first attempt to simulate cumulus convective mechanism is illustrated in Fig. 14. It was assumed that a convective transfer of moisture through the top of the boundary existed which was just sufficient to maintain the relative humidity at some critical value.

In experiments to establish the critical value for use in this convective mechanism, it was found that the θ_e at the eye wall was very sensitive to small variations. The percentage of specific humidity to be removed was varied along with the relative humidity criterion which initiates the mechanism. Figures 16 and 17 show that variation of either the percentage or relative humidity criterion caused a varying specific humidity content. The specific humidity is crucial in determining the equivalent potential temperature at the eye wall, θ_{ei}, which specifies the maximum wind speed, Eq. (17). Small modification procedures produced rather large variations in the intensity of the hurricane. Therefore specification of the critical relative humidity essentially determined the steady-state hurricane.

The first attempt at moisture reduction led to some improvement in the model. By preventing super-saturation, the moisture field was more representative. One of the objectives of the study was to achieve a steady-state condition in the hurricane when the ocean temperature was held fixed. However it was found that regardless of the initial maximum tangential wind speed, the predicted hurricane had the same intensity, as shown in Fig. 18. For experiments
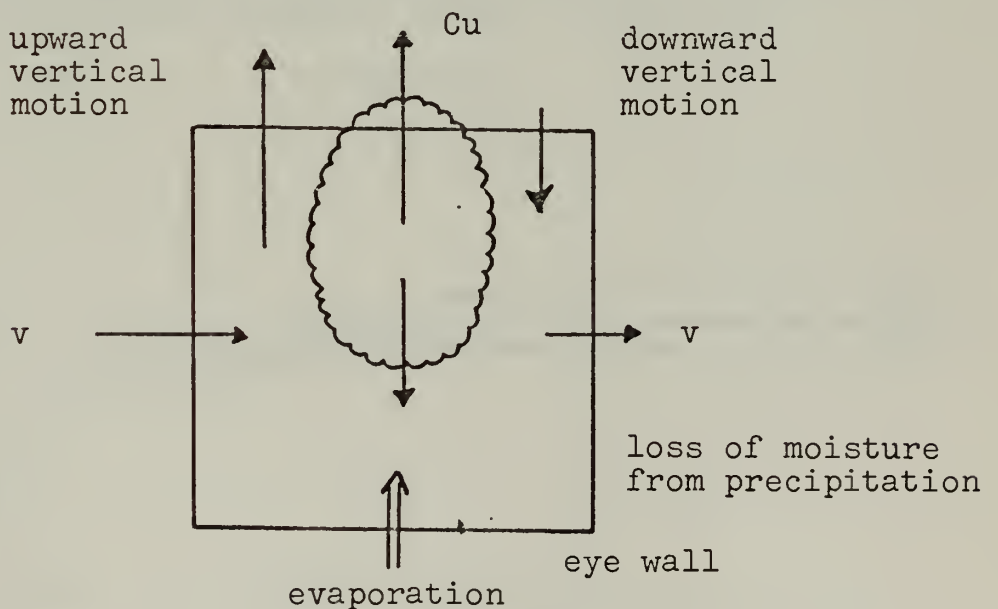
Convective moisture mechanism without Ta adjustment.
Note that a loss of moisture through top of boundary
layer occurs with ascent in cloud and compensating
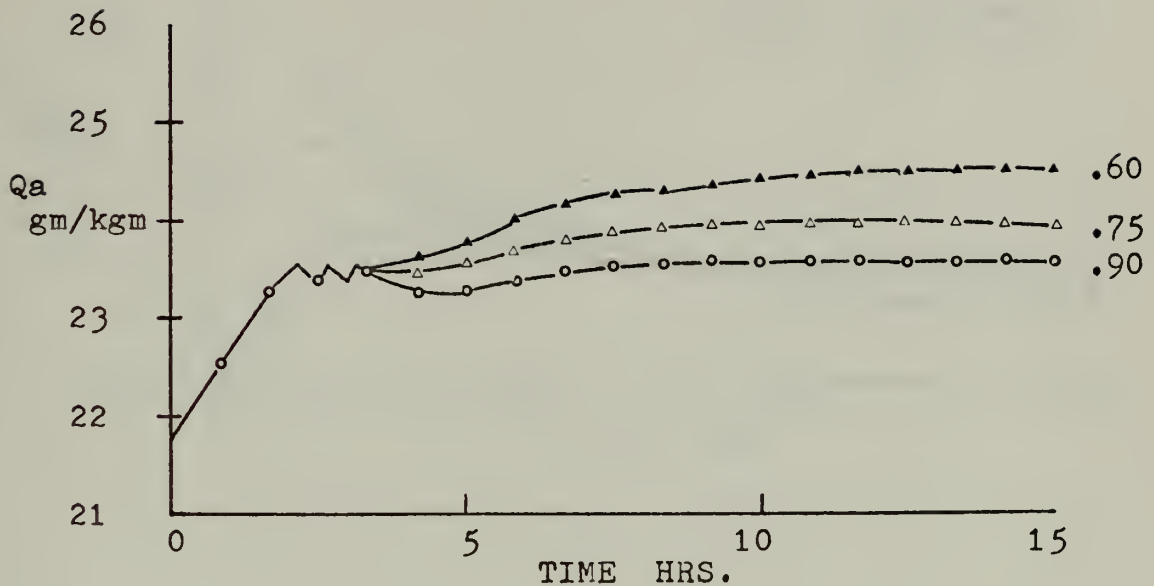descent in the environment.

Figure 14

Convective moisture mechanism with Ta adjustment.
Note that warming from the release of latent heat
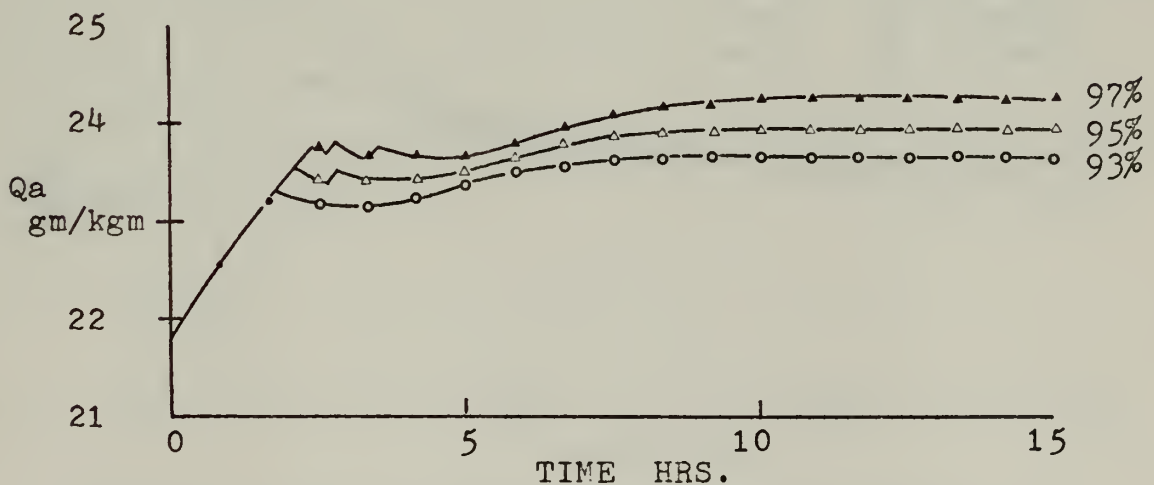occurs in boundary layer.

Figure 15

Specific humidity with varying amounts of convective
moisture reduction (fraction of 1% removal) for a
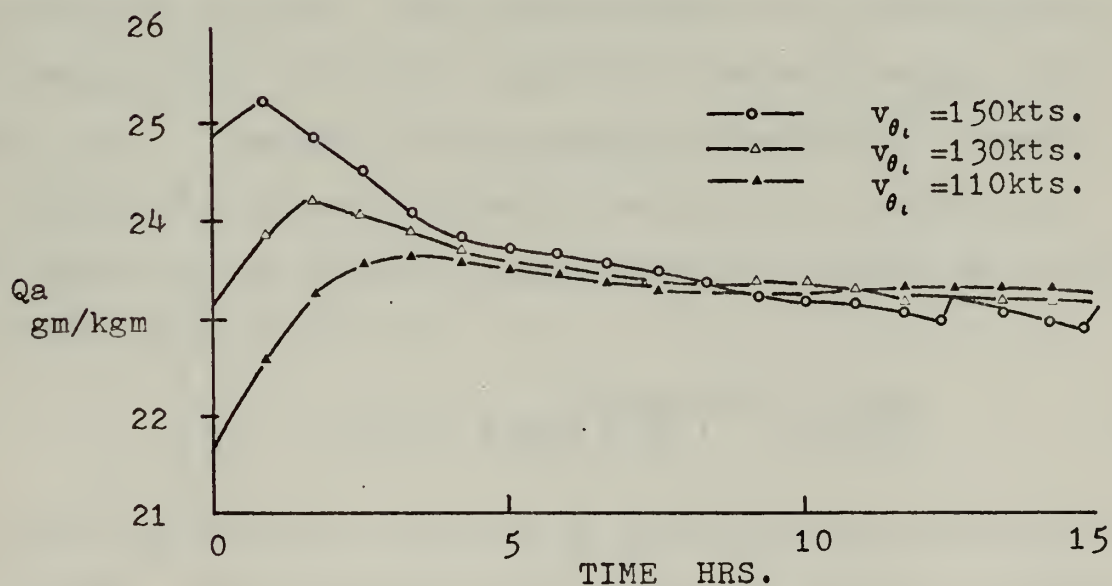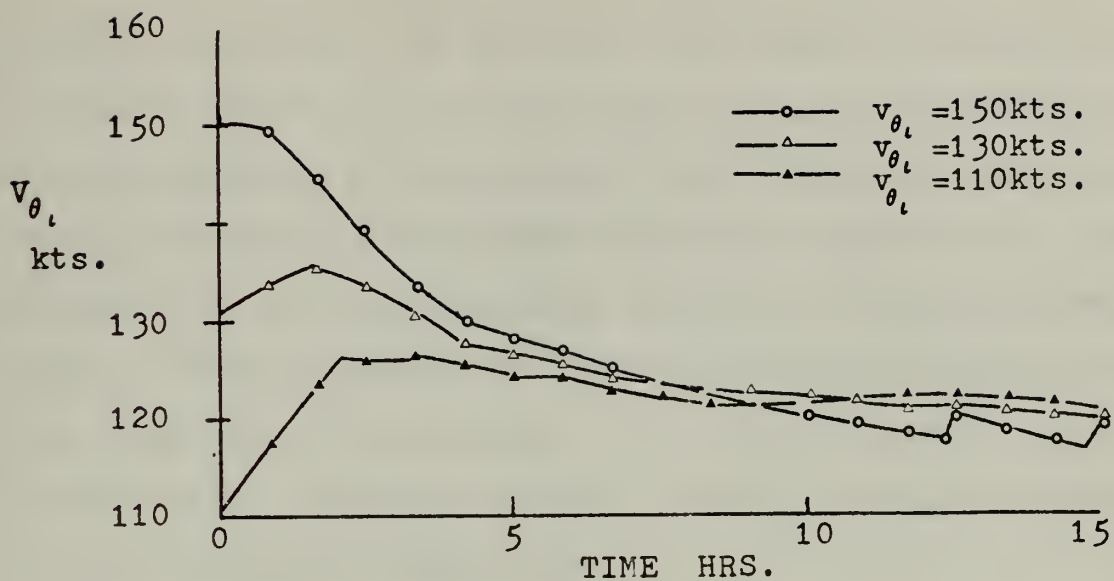critical relative humidity of 95%. (See text.)

Figure 16

Specific humidity with varying relative humidity cri-
teria for a convective moisture reduction of .75 of 1%.
(See text.)

Figure 17

Maximum tangential wind speed (upper diagram) and
specific humidity (lower diagram) for various initial
v_{θ_ι} values with a convective mechanism without a tempera-
ture adjustment.

Figure 18

with high initial wind speeds, and thus high specific hu-
midity values near the eye wall, the removal of moisture by
increased convection was not offset by an increased poten-
tial temperature of the surface air. Therefore there was a
steady decrease in equivalent potential temperature at the
eye wall, with a corresponding decrease in the wind speed,
until a balance between the convective flux of moisture and
the evaporation was achieved. This led to a second approach
to offset the decreased specific humidity with an increase
in the atmospheric temperature.

Recall that the equivalent potential temperature, Θ_e,
is conserved in a moist adiabatic process. In the second
approach (see Fig. 15), the percentage moisture reduction
is still utilized, but the process is assumed to be a psuedo-
adiabatic process. This modification was accomplished by
computing the equivalent potential temperature, Θ_e, from
Eq. (24). The equivalent potential temperature is main-
tained after the reduction of the specific humidity by
increasing the potential temperature of the air to a value
given approximately by,

$$T_a = -273.2 + \Theta_e \bigg/ \left(\frac{1000}{P}\right)^{.286} e^{\frac{L\, Q_a}{c_p\, T_a}} \qquad (25)$$

where the previous value of Ta is used in the denominator.
Yet Fig. 19 demonstrates that too much warming accompanied
the moisture decrease. The equivalent potential temperature
increased slowly, resulting in a corresponding growth in
maximum tangential wind speed. Therefore a second
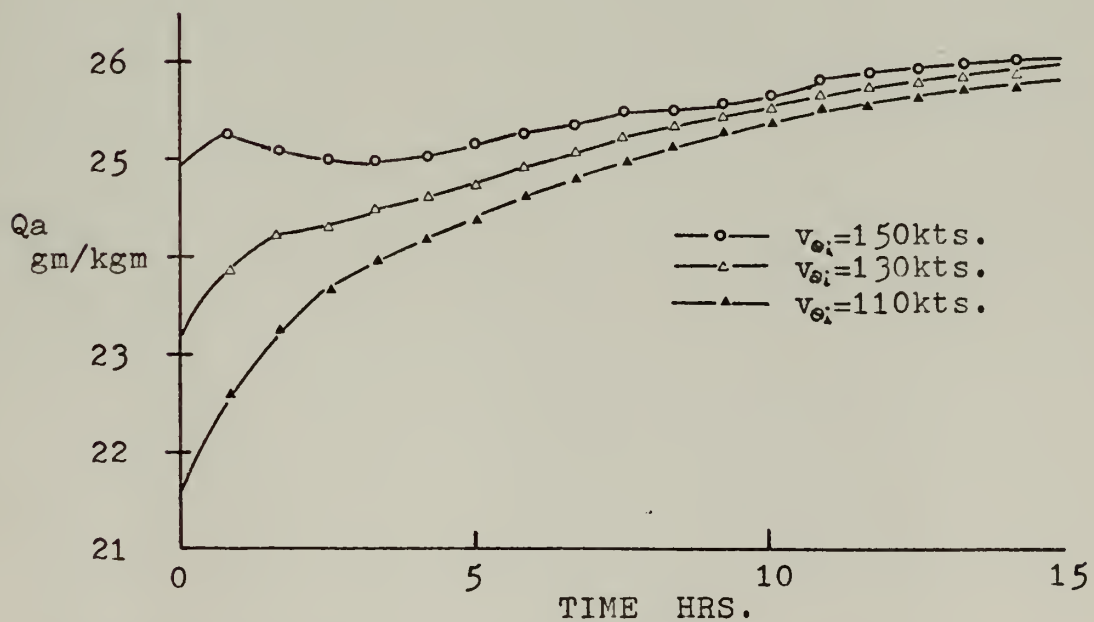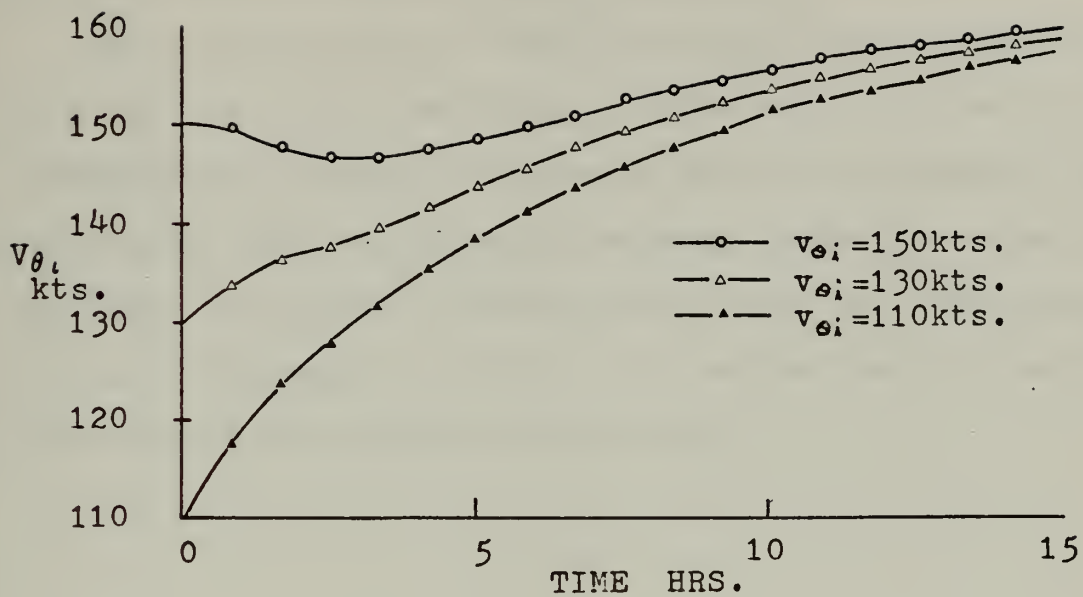
Maximum tangential wind speed (upper diagram) and
specific humidity (lower diagram) for various initial
$v_{\theta i}$ values with a convective mechanism with a tempera-
ture adjustment.

Figure 19

"steady-state hurricane" has been described for a different specification of the convective mechanism.

These experiments to reduce the predicted excess moisture point out the sensitivity of the model. Evidently a combination of moisture transport out of the boundary layer, along with some precipitation warming within the layer, is necessary to produce a steady-state hurricane. Experimentation is necessary to relate the combination of these two effects to the intensity of the storm.

V. OCEAN MODEL PROFILES

Two nearly steady-state models of the atmospheric flow, presented in the modification of the model section, allow comparison of the effect of hurricane circulation upon the ocean profile. The ocean temperature was allowed to vary, due to wind mixing.

A comparison of hurricane heat potential (HHP) and the changes in sea-surface temperature, Tw, will be made to show the effect of the two hurricanes. Hurricane heat potential is a function of the sensible and latent heat transfer as given by Eqs. (8) and (9). The change in this parameter represents the total energy loss experienced by the ocean due to the air-sea boundary fluxes. The greatest loss of heat potential occurred near the eye wall where the latent and sensible heat transfer·was the largest, as shown by Figs. 20 and 22. The accumulated energy loss increased with time with a loss of 1600-1900 cal/sq cm per day being accounted for by this approach. This energy loss is small compared to an atmospheric heat balance calculation of Malkus (1962), in which the total energy loss approached 3000 cal/sq cm per day. In the present model the hurricane heat potential calculation does not include radiation, or advection and upwelling effects.

Comparison of Figs. 20 and 22 indicates that the heat potential loss near the eye wall was less for the atmospheric
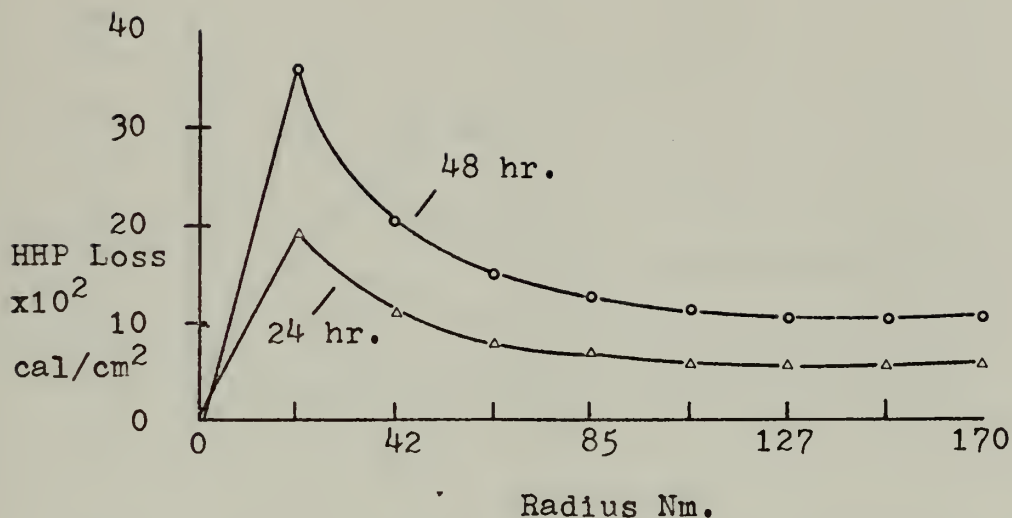
Hurricane heat potential loss for varying daily
observations for a convective mechanism without a
temperature adjustment.

Figure 20
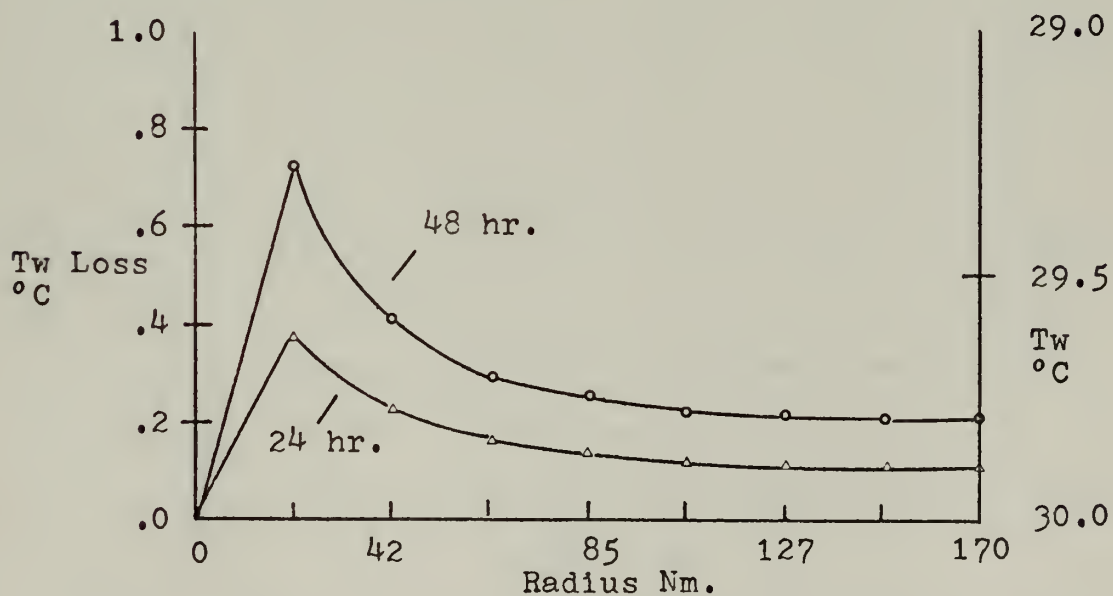
Sea-surface temperature loss for varying daily
observations for a convective mechanism without a
temperature adjustment.

Figure 21

Same as Fig. 20 except for convective mechanism with a temperature adjustment.

Figure 22

Same as Fig. 21 except for convective mechanism with a temperature adjustment.

Figure 23

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model with a convective mechanism with the temperature adjustment. This result was some what surprising since the wind speeds were larger in this model. The sensible and latent heat transfers near the eye wall were smaller because the air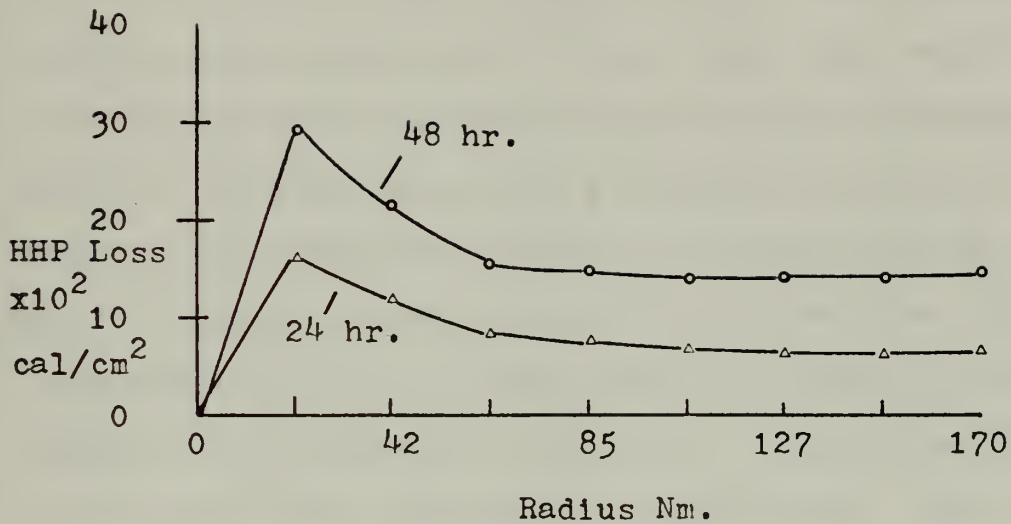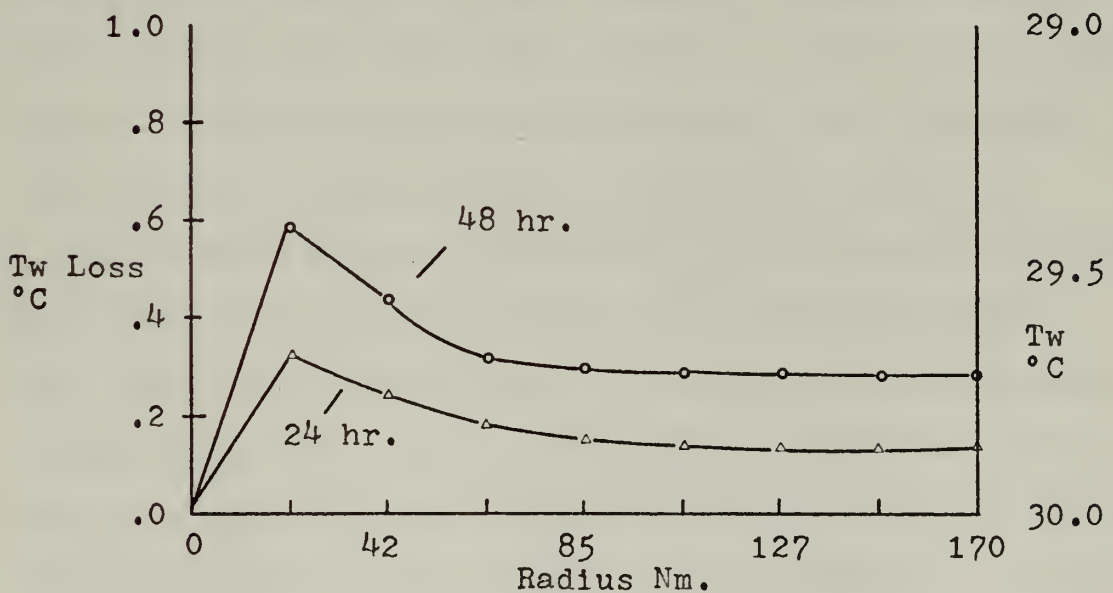-sea temperature and moisture differences were smaller. In this region where convective heating was important, the temperature tends to increase through precipitation warming. As the sea-surface temperature, Tw, decreased and the air temperature, Ta, increased, the sensible heat transfer was smaller. A similar explanation can be given for a reduction in evaporation. That is, the specific humidity, Qa, was larger (for the same critical relative humidity) in the adjusted temperature case because the saturation limit increases with temperature. Away from the eye wall, the wind speed became the dominant factor in the sensible and latent heat transfer equations. As shown in Fig. 22, the change in HHP was larger at these radii for the convective mechanism with temperature adjustment.

The simple model of calculating the sea-surface temperature from the loss of hurricane heat potential (HHP), Eqs. (15) and (16), resulted in a temperature field which cooled with time. Since the cooling occurs over a deep layer a nearly uniform cooling rate of .3 - .4C per day was noted near the eye wall region. This cooling rate is small compared with observed values of 1 - 2C cooling, as might be expected since radiation, horizontal advection in the ocean and upwelling effects have not been included. The resulting

sea-surface temperature fields point out the same features when comparing the two different convective mechanisms as were noted in the hurricane heat potential discussion.

Evolution of the maximum tangential wind speed over an extended period of integration is shown in Fig. 24. As noted previously the maximum tangential wind speed had a lower value without a temperature adjustment than when the surface temperature was adjusted for the effect of precipitation heating within the boundary layer. When the ocean temperature was allowed to vary, the resulting maximum wind speeds were slightly lower even in the first few hours, Fig. 24. As the ocean surface cooled, the sensible heat transfer decreased, and produced slightly cooler atmospheric temperatures at the eye wall. Again, restrictions on the relative humidity value reduced the specific humidity, resulting in a lower tangential wind component.
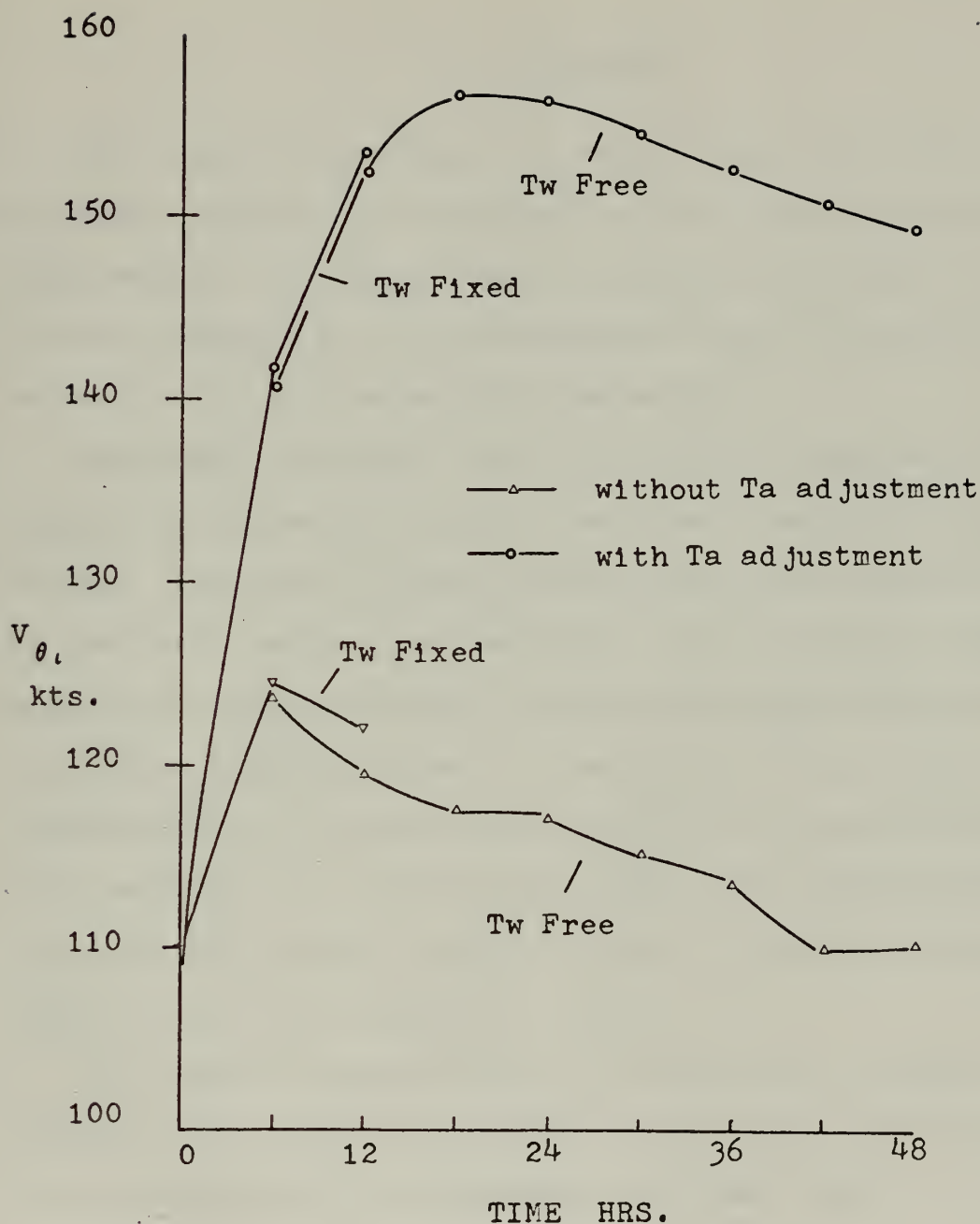
Comparison of maximum tangential wind speed with sea-surface temperature fixed (Tw fixed) and sea-surface temperature allowed to vary (Tw free) for the model incorporating a convective mechanism with and without temperature adjustment.

Figure 24

VI. CONCLUSION

This research utilized the steady-state hurricane model proposed by Riehl in experiments with a simple interacting ocean model. The purpose was to examine the effect of the air-sea exchange in a fixed boundary layer on the hurricane vortex and on the sea-surface temperature.

The model predicted fields of hurricane parameters which compare with observed surface features of mature tropical storms. The radial inflow profile within the boundary layer is realistic, with a corresponding vertical motion field similar to observed values. Because the model does not have a detailed boundary layer, the solution of the radial wind component due to surface stress is not very sophisticated. More resolution in the boundary layer and inclusion of differences in surface roughness would possibly produce a better radial wind component solution.

The simple representation of temperature advection plus sensible heat exchange produced the observed inward decrease in temperature in the atmospheric boundary layer. However an excessive accumulation of moisture occurred in the boundary layer. Two attempts to correct for this situation have shown that small changes in the surface moisture and temperature values strongly affect the maximum wind speed predicted. The proportion of the water vapor transported out of the top of the boundary layer without adding latent

heat to the boundary layer appears to be an important factor in determining the intensity of a steady-state hurricane.

Although the ocean model is quite simple, realistic horizontal oceanic temperature profiles with respect to the hurricane center were obtained. However the magnitude of changes in oceanic heat content and the sea-surface temperatures were smaller than expected in actual hurricanes, since advection and upwelling effects were not included in the ocean model. A steady-state cannot be reached in a stationary storm because of the decreasing sea-surface temperature field. Provision for movement of the hurricane with respect to the ocean would allow prediction of the intensity of the atmospheric system and the oceanic effects. Improvement of the oceanic model, to include a combination of vertical mixing and upwelling, would also give a more realistic view of the consequences of air-sea interaction below a hurricane.

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13. ABSTRACT

The equations developed by Riehl (1963) for a steady, symmetrical hurricane are used to specify a vortex over a fixed depth boundary layer, which interacts with a simple ocean model. Only vertical mixing in the surface layer is included in the oceanic portion of the model. Interaction of the atmosphere and the ocean occurs through the prediction of boundary layer parameters. Comparison of the predicted boundary parameters with observed mature tropical storms shows good agreement for all variables except the specific humidity, which exceeds saturation. Two attempts to correct for this situation have shown the proportion of the water vapor transported out of the top of the boundary layer without adding latent heat to the boundary layer appears to be an important factor in determining the intensity of a steady-state hurricane.

Although the ocean model is quite simple, realistic horizontal oceanic temperature profiles with respect to the hurricane center were obtained.

KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
hurricane						
atmospheric model						
oceanic model						